Dear Editor,

Thank you for giving us the opportunity to submit a revised draft of the manuscript “Numerical integration scheme for coupled elastoplastic-viscoplastic constitutive law for tunnels” for publication in the International Journal of Geomechanics. We appreciate the time and effort that you and the reviewers dedicated to providing feedback on our manuscript and are grateful for the insightful comments on and valuable improvements to our paper. We have incorporated most of the suggestions made by the reviewers. Please see below, in blue, for a point-by-point response to the reviewers’ comments and concerns. All page numbers refer to the revised manuscript file.

**REPLIES TO EDITOR’S E-MAIL**

• **Please note the byline on your manuscript text doesn’t match the manuscript data entered (add/edit/delete authors step). Please correct this and resubmit.**

The name in the system was changed from Felipe Quevedo to Felipe Pinto da Motta Quevedo as in the article.

• **Please upload your revised manuscript file in Microsoft Word or LaTex format. If you using LaTex, you may submit a PDF file for review. Please see our LaTex instructions on the Author main page for more information.**

The revised manuscript is submitted in pdf with the project files in Latex.

• **Remove the figures from your manuscript text and upload them separately (one figure per file) in TIFF, EPS or PDF format. Also, please make sure to reference the figure number in each file name.**

The figures were removed from the manuscript and registered separately in the system.

• **Double-spaced list of figure captions. Please provide a double-spaced list of figure captions with your submission. This can be at the end of your manuscript text or uploaded as a separate Word file. Also, please make sure if you have figures labeled as Figure 1a, 1b, etc. that the captions for these parts of the figure are included in your Figure Caption List.**

The list of figures is at the end of the manuscript and is formatted according to the Latex template.

• **Embedded Tables. Please remove tables from within the text of your paper and place them at the end of your manuscript after the references . If you upload them separately, please make sure they are uploaded in Microsoft Word/LaTex format.**

Tables are at the end of the manuscript after the references.

**Also, please note in order to clarify math for copyeditors, please ensure that you use boldface for matrices, vectors, and tensors; italics for all variables and lowercase Greek letters; and roman for all numerals, uppercase Greek characters, and mathematical operators.**

It’s already correct.

**Please submit the revised manuscript and a detailed response to the reviewers' criticisms by logging onto the Editorial Management system at https://www.editorialmanager.com/jrngmeng/ and clicking on the "Submissions Needing Revision" link.**

Ok.

**REPLIES TO REVIEWER 1**

**This is very interesting research work to apply the hyperelasticity to the geo-material. In addition, this is the suitable method to employ this constitutive law to the integral of the implicit method. However, it is difficult to understand it. The authors should friendly revise some explanation in each model's part.**

**1. The reviewer does not think that it is necessary to explain according to Hyperelasticity. However, the authors should explain it more concisely so that the reader can understand it. Why did the authors employ Hyperelasticity to the constitutive model of the geo-material. The authors should explain the reason in detail.**

This is not a study considering hyperelasticity. So, the equations that express the specific free energy were removed.

**2. The potential function of hyperelasticity, ψe, should be described.**

This is not a study considering hyperelasticity. So, the equations that express the specific free energy were removed.

**3. The authors should declare that the cohesion, c, is a variable when it is employed at the yield function.**

Okay. It’s done.

**4. The authors should show the concrete form of the plastic potential function. In addition, they should explain why they use such a function.**

Okay. Added the following sentence in line 102:

“[…] For Drucker-Prager potential flow  has , , . This potential function has a numerical advantage due to its smoothness. Another advantage is that can simulate the volume variation during the evolution of plastic deformations. This effect is commonly introduced through unassociated plasticity, adopting, instead of the friction angle a dilatancy angle in the potential function .”

**5. There are many variables in the hardening rule, and the authors should explain them such as the magnitude relationship of cp, ci, cr, etc., in detail. In addition, the authors should also explain the difference between zones. It should also show a comparison with the response of the actual material.**

Okay. It’s done. Figure 1 has been added showing the zones and the comparison with triaxial material test.

**6. The reviewer thinks it is better to explain the validity of the constitutive rule by showing some analysis examples (stress-strain relations) such as triaxial tests. At that time, it is better to compare it with the actual experimental data.**

The Drucker-Prager model is a classic model used for geomaterials. Figure 1 is added, which shows the results of the model with a triaxial test of the Ductil Boom Clay Rock Mass.

**7. Equations (2), (3), (4): The parameter, q, in the proposed model may be variable. In essence, it is a model that the cohesion, c, changes due to plastic deformation. However, it is difficult to understand. The authors should describe and explain the yield function and the cohesion in the yield function.**

Okay. It’s done.

**8. Equation (5): The reviewer thinks that it is better to remove equation (5) since it is difficult to understand it.**

Okay. It’s done.

**9. Equation 6: The authors indicated the Load angle, θ. However, it was not employed in the yield function. On the other hand, it was employed in the plastic potential. The authors should explain it.**

The flow function is written in the general form, where all the stress invariants commonly used to write the yield surfaces appear. In the sequence,  does not appear because the Drucker-Prager surface is independent of this stress invariant. Added the following sentence in line 87:

“The Lode angle does not appear because the Drucker-Prager surface does not depend on this angle in the deviator plane”.

The potential function is also described in its general form, however, in the sequence, the Drucker-Prager potential function does not depend on this term with .

**10. In the geomechanics, the compression is often implicitly positive. At the first appearance, σ, the authors may declare the tension is positive.**

The follow sentence, below the first appearance the constitutive relationship, in line 65, was added:

“where  .is the stress tensor (positive compression convention),  and  are fourth-order tensors representing the elastic and elastoplastic modulus, respectively.”

**11. Equation 10: The formula of the function, g, should be described.**

It’s done. See question 4.

**12. Equation 11: g3 should be solved and the formula of dJ3/dσ should be also described.**

It’s done. [Pedir para o Samir conferir]

**13. Equation 24: The first term of the left hand side is the strain rate. Therefore, "dot" is necessary.**

It’s done.

**14. Equation 26: It is normally use to ε = εe+εvp. It is rear to separate between plastic strain, εp, and εvp. The authors should explain how εp and εvp were calculated individually.**

The models are coupled in series by adding the corresponding deformations. The individual calculation of each is described separately. The following paragraph, in line 181, has been changed:

“The proposed elastoplastic-viscoplastic model is constructed by the serial association of the constitutive models described above, i.e., , which leads to the following constitutive relationship:



This association can be seen in the one-dimensional representation of Fig. 2. […]”

**REPLIES TO REVIEWER 2**

**The concepts presented in the paper are available in standard texts. May focus on the work done by the Authors using UPF.**

**Elastoplastic-viscoplastic constitutive formulations are presented in the paper. Generally, such coupled analysis is required if rate-independent (instantaneous) analysis needs to be augmented with time-dependent yielding analysis such as the effect of creep. The concept presented in the paper is well documented in standard literature (both rate-independent and time-dependent). Coupling of these two methods are also presented in several literatures as also mentioned by the authors. In that sense, the paper does not provide any new information.**

Although each separate model is well documented in the literature; the coupling models are rare and very specific to the surfaces used. A flowchart for a general scheme is rarely presented for these coupled models.

**However, a coupling integration algorithm is presented in UPF platform of ANSYS which may have some interest to the readers of this journal. It is encouraged that the authors focus their paper in that direction rather than elaborating the concept already available in the standard books and literature. While revising the paper, authors may highlight the following:**

**1. Dilation of rock materials is neglected in the present study. Dep in elasto-plastic analysis will be unsymmetrical in non-associative flow. Please make comments on symmetrisation techniques or comment whether one needs a non-symmetrical solver. Authors may refer to the following symmetrisation papers.**

**Pande et al., 1986, “Symmetric tangential stiffness formulation for non-associative plasticity”, Compu Geotech, 2(2) 89-99**

**Deb et al., 2013, “Generalized symmetric formulation of tangential stiffness for nonassciative plasticity”, J. of Engg. Mech, Vol 139, issue 2.**

The effect of dilation is considered in the algorithm. About the non-symmetric solver, it was added in line 257:

“Updating the constitutive modulus increase the convergence rate of equilibrium iterations. But requires the assembly of the global stiffness matrix at each equilibrium iteration, which has a computational cost for large systems. Furthermore, in unassociated plasticity, the constitutive modulus is not symmetric, and its update leads to a non-symmetric global stiffness matrix, requiring a non-symmetric solver for the global equilibrium iterations. Constitutive module symmetrization techniques, as in Deb et al. (2013), also can be used. However, the algorithm converges even not updating the constitutive modulus. Due the computational costs, this update is optional.”

**2. Express momentum balance equation in static condition and make comments on the increment of external load, especially whether it will be time dependent or not. Generally, for elasto-plastic analysis delta(t) is a pseudo-parameter, however, it is an important parameter for visco-plasticity. Will the stress corrections in elasto-plastic analysis now be depended on delta(t)?**

The authors do not think that it is necessary to express the balance of moments equation, since we are working specifically on the constitutive equations of the material. However, the following paragraph has been added, in line 314:

“In elastoplasticity, time is just a pseudo-parameter to mark the load history and control the incremental solution, but it does not influence the constitutive relationship. In coupled model, elastoplasticity becomes time-dependent since as it depends on viscoplasticity.”

**3. Authors have assumed that increment of strain(vp) will be estimated first and stress will be updated before elasto-plastic analysis starts. One would think it may happen in the reverse way.**

To make the sequence clear, the following paragraph has been modified in line 305:

“As viscoplasticity is integrated through a semi-implicit rule in which all variables are calculated with known stresses (from substep ), the viscoplastic strain increment is calculated first. Subsequently, it is descounted from the total strain increment of the elastic predictor in the elastoplasticity algorithm. This is the correct sequence for the coupling, since elastoplasticity algorithm update the stress state for substep  in order verify the criterion . If the plastic strain were calculated before the viscoplastic strain, the criterion . […]”

**4. Define ci, cp, cr in equation 13.**

Okay. It’s done in line 127. In Addition, Figure 1 has been added showing the zones and the comparison with triaxial material test.

**5. Elaborate on the UPF code in ANSYS for the benefit of the readers. This is probably the novelty of this paper. Authors may schematically present the code block in Fortran. Title of the paper may be changed accordingly.**

The code was added to a repository and linked in the article. The implementation sequence is shown in the flowchart in Figure 4. The idea of presenting the flowchart instead of focusing on the Fortran code implementation is that the model can be implemented in any software and any programming language. And, in this way, it will not be restricted to ANSYS users and Fortran 77 readers.

**6. Is superscript p valid in equation given in line 214?**

Okay. It’s done.

**7. The example problem is solved considering associative flow. Analyze the same example considering non-associative flow rule.**

Okay. It’s done. The Figure 8 show this analysis.

**8. Line 318: ratio not ration.**

Okay. It’s done.

**The paper needs major revision as mentioned above.**

**REPLIES TO REVIEWER 3**

**1. Authors should review the consistency and notation of all equations.**

Okay. It’s done.

**2. The "zones" in equations 13 and 15 are not well explained (Authors could make a graphic showing these different zones)**

Okay. It’s done. Figure 1 has been added showing the zones and the comparison with triaxial material test.

**3. The literature review should be reduced and more details should be given on the coupling of the constitutive models**

Since the coupled model is the junction of two models (elastoplastic and viscoplastic) it is essential to present the coupled model through these models. Anyway, in addition to the flowchart in Figure 4, the following details about the coupled model have been added:

In line 305:

“As viscoplasticity is integrated through a semi-implicit rule in which all variables are calculated with known stresses (from substep ), the viscoplastic strain increment is calculated first. Subsequently, it is descounted from the total strain increment of the elastic predictor in the elastoplasticity algorithm. This is the correct sequence for the coupling, since elastoplasticity algorithm update the stress state for substep  in order verify the criterion . If the plastic strain were calculated before the viscoplastic strain, the criterion . […]”

In line 304:

“In elastoplasticity, time is just a pseudo-parameter to mark the load history and control the incremental solution, but it does not influence the constitutive relationship. In coupled model, elastoplasticity becomes time-dependent since as it depends on viscoplasticity.”

In line 189:

“An interesting aspect of this coupled model, using Drucker-Prager criterion for  and  with , is that cohesions controls the solution regime. If  and  the solution is purely elastic. If only  we have a purely elastoviscoplastic solution, and if only  we have a purely elastoplastic solution.”

**4- The ANSYS APDL script for the FEM model and the USERMAT subroutine in FOTRAN for the rock constituent model should be publicly available datasets. It is recommended to use for example "Datasets related to this article can be found at [INSERT PERMANENT URL(s) TO BE LINKED TO DATASET], hosted at [NAME OF HOSTING REPOSITORY] ([CITATION TO DATASET])".**

Okay. It’s done in line 312. In addition the paper has the following data availability statement:

“Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. (ANSYS APDL script for FEM model and USERMAT subroutine in FORTRAN 77 for constitutive rock mass model).”

[Ver como criar um repositório para os arquivos.]